# Mastering Differential Equations

### **Part I: Introduction to Differential Equations**

**1. Introduction to Differential Equations**

* 1.1. What is a Differential Equation?
* 1.2. Types of Differential Equations
* 1.3. Order and Degree
* 1.4. Linear vs. Nonlinear Equations
* 1.5. Applications of Differential Equations
* 1.6. Basic Terminology and Concepts

**2. First-Order Differential Equations**

* 2.1. Separable Equations
* 2.2. Linear First-Order Equations
* 2.3. Exact Equations
* 2.4. Integrating Factors
* 2.5. Homogeneous Equations and Substitutions
* 2.6. Bernoulli Equations
* 2.7. Applications of First-Order ODEs

**3. Second-Order Linear Differential Equations**

* 3.1. Homogeneous Equations with Constant Coefficients
* 3.2. The Characteristic Equation
* 3.3. Repeated and Complex Roots
* 3.4. Nonhomogeneous Equations
* 3.5. Method of Undetermined Coefficients
* 3.6. Variation of Parameters
* 3.7. Applications of Second-Order ODEs\*\*

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### **Part II: Higher-Order and Systems of ODEs**

**4. Higher-Order Linear Differential Equations**

* 4.1. Homogeneous Higher-Order Equations
* 4.2. Nonhomogeneous Higher-Order Equations
* 4.3. Method of Undetermined Coefficients for Higher Orders
* 4.4. Variation of Parameters for Higher Orders
* 4.5. Applications

**5. Systems of Differential Equations**

* 5.1. Introduction to Systems
* 5.2. Linear Systems with Constant Coefficients
* 5.3. Matrix Methods
* 5.4. Eigenvalues and Eigenvectors
* 5.5. Diagonalization
* 5.6. Phase Plane Analysis
* 5.7. Stability of Equilibrium Points
* 5.8. Nonlinear Systems and Linearization
* 5.9. Applications of Systems of ODEs

**6. Series Solutions of Differential Equations**

* 6.1. Power Series Solutions
* 6.2. Radius of Convergence
* 6.3. Frobenius Method
* 6.4. Bessel’s Equation
* 6.5. Legendre’s Equation
* 6.6. Special Series Solutions

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### **Part III: Laplace Transforms and Integral Equations**

**7. Laplace Transforms**

* 7.1. Definition and Properties
* 7.2. Inverse Laplace Transforms
* 7.3. Solving Initial Value Problems
* 7.4. Laplace Transforms of Special Functions
* 7.5. Applications to Differential Equations

**8. Integral Equations**

* 8.1. Introduction to Integral Equations
* 8.2. Types of Integral Equations
* 8.3. Solving Integral Equations
* 8.4. Applications

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### **Part IV: Partial Differential Equations (PDEs)**

**9. Introduction to Partial Differential Equations**

* 9.1. Definition and Classification
* 9.2. Examples and Applications

**10. Classification of Partial Differential Equations**

* 10.1. Linear vs. Nonlinear PDEs
* 10.2. Second-Order PDEs
* 10.3. Elliptic, Parabolic, and Hyperbolic Equations

**11. Method of Separation of Variables**

* 11.1. Separation in Cartesian Coordinates
* 11.2. Separation in Polar Coordinates
* 11.3. Solving Boundary Value Problems

**12. Fourier Series and Fourier Transforms**

* 12.1. Fourier Series
* 12.2. Convergence of Fourier Series
* 12.3. Fourier Transforms
* 12.4. Applications to PDEs

**13. Sturm-Liouville Theory**

* 13.1. Sturm-Liouville Problems
* 13.2. Orthogonality of Eigenfunctions
* 13.3. Expansion in Eigenfunctions
* 13.4. Applications

**14. Green’s Functions**

* 14.1. Introduction to Green’s Functions
* 14.2. Construction of Green’s Functions
* 14.3. Applications to Linear PDEs

**15. Numerical Methods for Partial Differential Equations**

* 15.1. Finite Difference Methods
* 15.2. Finite Element Methods
* 15.3. Method of Characteristics
* 15.4. Stability and Convergence

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### **Part V: Advanced Topics in Differential Equations**

**16. Nonlinear Differential Equations**

* 16.1. Introduction to Nonlinear ODEs
* 16.2. Exact Solutions
* 16.3. Qualitative Analysis
* 16.4. Phase Plane Methods
* 16.5. Limit Cycles and Bifurcations

**17. Dynamical Systems and Chaos Theory**

* 17.1. Introduction to Dynamical Systems
* 17.2. Fixed Points and Stability
* 17.3. Bifurcation Theory
* 17.4. Chaos and Strange Attractors
* 17.5. Applications to Real-World Systems

**18. Perturbation Methods**

* 18.1. Introduction to Perturbation Theory
* 18.2. Regular and Singular Perturbations
* 18.3. Multiple Scales Analysis
* 18.4. Applications to Differential Equations

**19. Functional Analysis in Differential Equations**

* 19.1. Banach and Hilbert Spaces
* 19.2. Operators and Spectral Theory
* 19.3. Existence and Uniqueness Theorems
* 19.4. Sobolev Spaces
* 19.5. Applications to PDEs

**20. Special Topics**

* 20.1. Lie Groups and Symmetry Methods
* 20.2. Integrable Systems
* 20.3. Hamiltonian and Lagrangian Formulations
* 20.4. Soliton Theory
* 20.5. Fractional Differential Equations

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### **Part VI: Applications of Differential Equations**

**21. Applications in Physics**

* 21.1. Mechanics
* 21.2. Electromagnetism
* 21.3. Quantum Mechanics
* 21.4. Relativity

**22. Applications in Engineering**

* 22.1. Control Theory
* 22.2. Electrical Circuits
* 22.3. Structural Analysis
* 22.4. Fluid Mechanics

**23. Applications in Biology and Medicine**

* 23.1. Population Dynamics
* 23.2. Epidemiology
* 23.3. Neural Networks
* 23.4. Pharmacokinetics

**24. Applications in Economics and Social Sciences**

* 24.1. Economic Modeling
* 24.2. Game Theory
* 24.3. Network Dynamics
* 24.4. Behavioral Models

**25. Computational Differential Equations**

* 25.1. Numerical Solution Techniques
* 25.2. Software and Tools
* 25.3. Simulation and Modeling

**26. Future Directions and Research in Differential Equations**

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### **Appendices**

**A. Mathematical Preliminaries**

* A.1. Linear Algebra Review
* A.2. Complex Analysis
* A.3. Calculus Refresher
* A.4. Vector Calculus

**B. Tables of Integrals and Transforms**

**C. Solutions to Selected Problems**

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